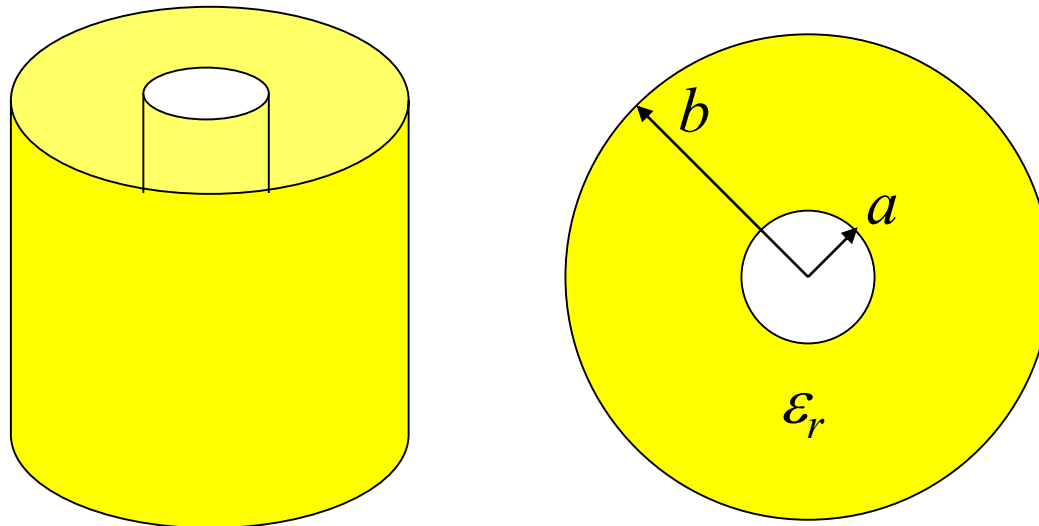


# COMSOLによる同軸線路の解析 ～ 数値ポート ～



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## Cの計算

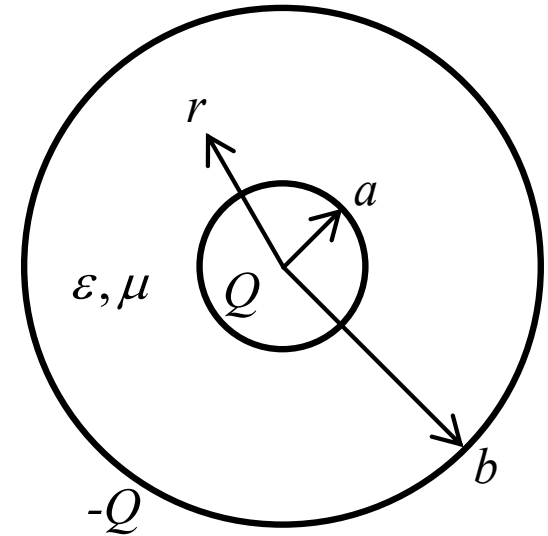
$$\oiint_S \mathbf{D} \cdot d\mathbf{S} = \iiint_V \rho dV \quad \text{ガウスの法則}$$

$$\varepsilon E_r \cdot 2\pi r = Q$$

$$E_r = \frac{Q}{2\pi\varepsilon r}$$

$$V = \int_a^b E_r dr = \frac{Q}{2\pi\varepsilon} [\log r]_a^b = \frac{Q}{2\pi\varepsilon} \log \frac{b}{a}$$

$$C = \frac{Q}{V} = \frac{2\pi\varepsilon}{\log \frac{b}{a}}$$



## Lの計算

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \iint_S \mathbf{i} \cdot d\mathbf{S} + \iint_S \frac{d\mathbf{D}}{dt} \cdot d\mathbf{S} \quad \text{アンペアの法則}$$

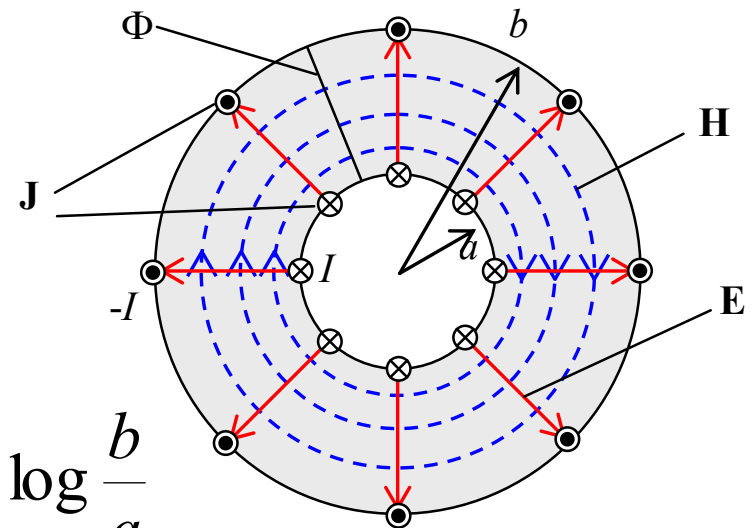
$$H_\varphi \cdot 2\pi r = I \quad H_\varphi = \frac{I}{2\pi r}$$

$$\frac{\Phi}{\mu} = \int_a^b H_\varphi dr = \frac{I}{2\pi} \log \frac{b}{a}$$

$$L = \frac{\Phi}{I} = \frac{\mu}{2\pi} \log \frac{b}{a}$$

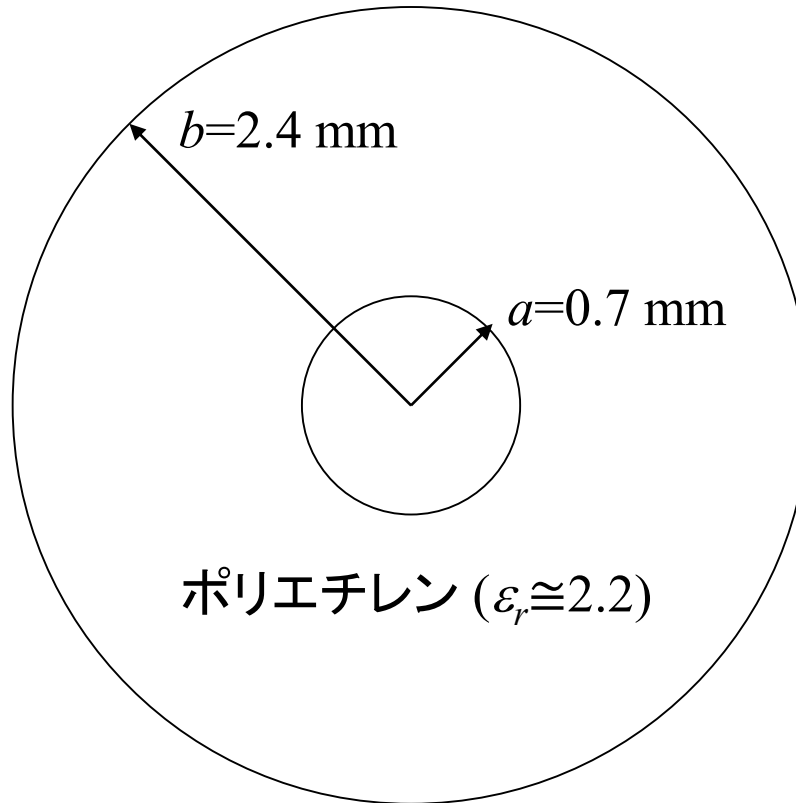
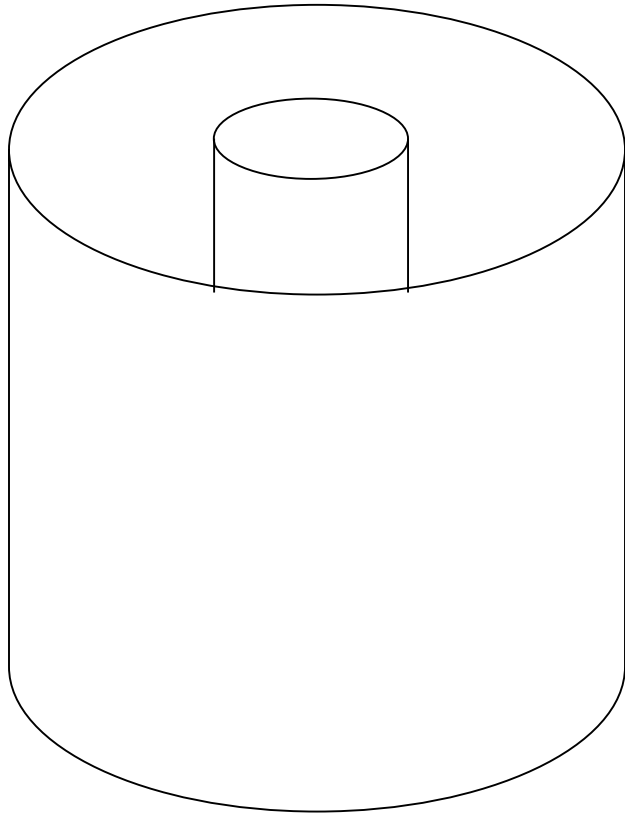
特性インピーダンス

$$Z = \sqrt{\frac{L}{C}} = \sqrt{\frac{\mu}{\varepsilon}} \frac{\log \frac{b}{a}}{2\pi}$$



# 同軸線路

JIS規格: 5D-2V同軸ケーブル



特性インピーダンス  $50 \Omega$

```

In[1]:= a = (1.4 / 2) * 10-3;
        b = a + 1.7 * 10-3;
        εr = 2.2;
        μr = 1.;

In[5]:= ε0 = 8.85418782 * 10-12;
        μ0 = 1.25663706 * 10-6;
        ε = εr * ε0;
        μ = μr * μ0;

        cap =  $\frac{2 * \pi * \epsilon}{\text{Log}\left[\frac{b}{a}\right]}$ 

        ind =  $\frac{\mu}{2 * \pi} \text{Log}\left[\frac{b}{a}\right]$ 
              対数

        z0 =  $\sqrt{\frac{\text{ind}}{\text{cap}}}$ 

Out[9]= 9.93322 * 10-11

Out[10]= 2.46429 * 10-7

Out[11]= 49.8082
  
```

coax\_highter\_order.mph - COMSOL Multiphysics

File Home Definitions Geometry Materials Physics Mesh Study Results Developer

Model Builder

- coax\_highter\_order.mph (root)
  - Global Definitions
    - Parameters
  - Materials
  - Component 1 (comp1)
    - Definitions
    - Geometry 1
      - Cylinder 1 (cyl1)
      - Cylinder 2 (cyl2)
      - Difference 1 (dif1)
      - Form Union (fin)
    - Materials
    - Electromagnetic Waves, Frequency
      - Wave Equation, Electric 1
        - Perfect Electric Conductor 1
        - Initial Values 1
        - Port 1
        - Port 2
    - Mesh 1
    - Study 1
      - Step 1: Boundary Mode Analysis 1
      - Step 2: Boundary Mode Analysis 2
      - Step 3: Frequency Domain
    - Solver Configurations
    - Results
      - Data Sets
      - Views
      - Derived Values
      - Tables
      - Electric Field (emw)
        - Surface 1
        - Arrow Surface 1
        - Arrow Surface 2
      - Export
      - Reports

Settings Properties

Parameters

Name	Expression	Value	Description
a	0.7[mm]	7E-4 m	
b	2.4 [mm]	0.0024 m	
len	10 [mm]	0.01 m	

Graphics Convergence Plot 1 Convergence Plot 2

Messages Progress Log Table 1

1.24 GB | 1.45 GB



# 材料(誘電体)の定義

coax\_higher\_order.mph - COMSOL Multiphysics

File Home Definitions Geometry Materials Physics Mesh Study Results Developer

Model Builder

- coax\_higher\_order.mph (root)
  - Global Definitions
    - Parameters
    - Materials
  - Component 1 (comp1)
    - Definitions
    - Geometry 1
      - Cylinder 1 (cyl1)
      - Cylinder 2 (cyl2)
      - Difference 1 (dif1)
      - Form Union (fin)
    - Materials
      - Dielectric (mat2)
        - Basic
    - Electromagnetic Waves, Frequency
      - Wave Equation, Electric 1
      - Perfect Electric Conductor 1
      - Initial Values 1
      - Port 1
      - Port 2
    - Mesh 1
    - Study 1
      - Step 1: Boundary Mode Analysis 1
      - Step 2: Boundary Mode Analysis 2
      - Step 3: Frequency Domain
    - Solver Configurations
    - Results
      - Data Sets
      - Views
      - Derived Values
      - Tables
      - Electric Field (emw)
        - Surface 1
        - Arrow Surface 1
        - Arrow Surface 2
      - Export
      - Reports

Settings Properties

Property Group

Label: Basic

Output Properties and Model Inputs

Quantities

- Model Inputs
- Output Properties
  - Absorption Coefficient
  - Activation Energy
  - Bulk Viscosity
  - Characteristic Acoustic Impedance
  - Coefficient of Hygroscopic Swelling
  - Coefficient of Thermal Expansion
  - Compressibility of Fluid
  - Density
  - Diffusion Coefficient

Property	Variable	Expression	Unit
Electrical conductivity	sigma ;...	0	S
Relative permeability	mur ; m...	1	1
Relative permittivity	epsilon_r...	2.2	1

Output properties

Expression:

Model inputs

Physical quantity Variable

Graphics

Convergence Plot 1 Convergence Plot 2

Messages Progress Log Table 1

1.26 GB | 1.45 GB



## 境界条件の設定

The screenshot displays the COMSOL Multiphysics interface for a model named 'coax\_higher\_order.mph'. The left sidebar shows the 'Model Builder' with a tree view including 'Component 1 (comp1)' and 'Study 1'. The central 'Properties' window is set to 'Perfect Electric Conductor' with 'Boundary Selection' set to 'All boundaries'. The right 'Graphics' window shows a 3D view of a coaxial cable structure with dimensions in mm. The bottom status bar indicates memory usage of 1.25 GB | 1.44 GB.

外導体・内導体をPECに設定



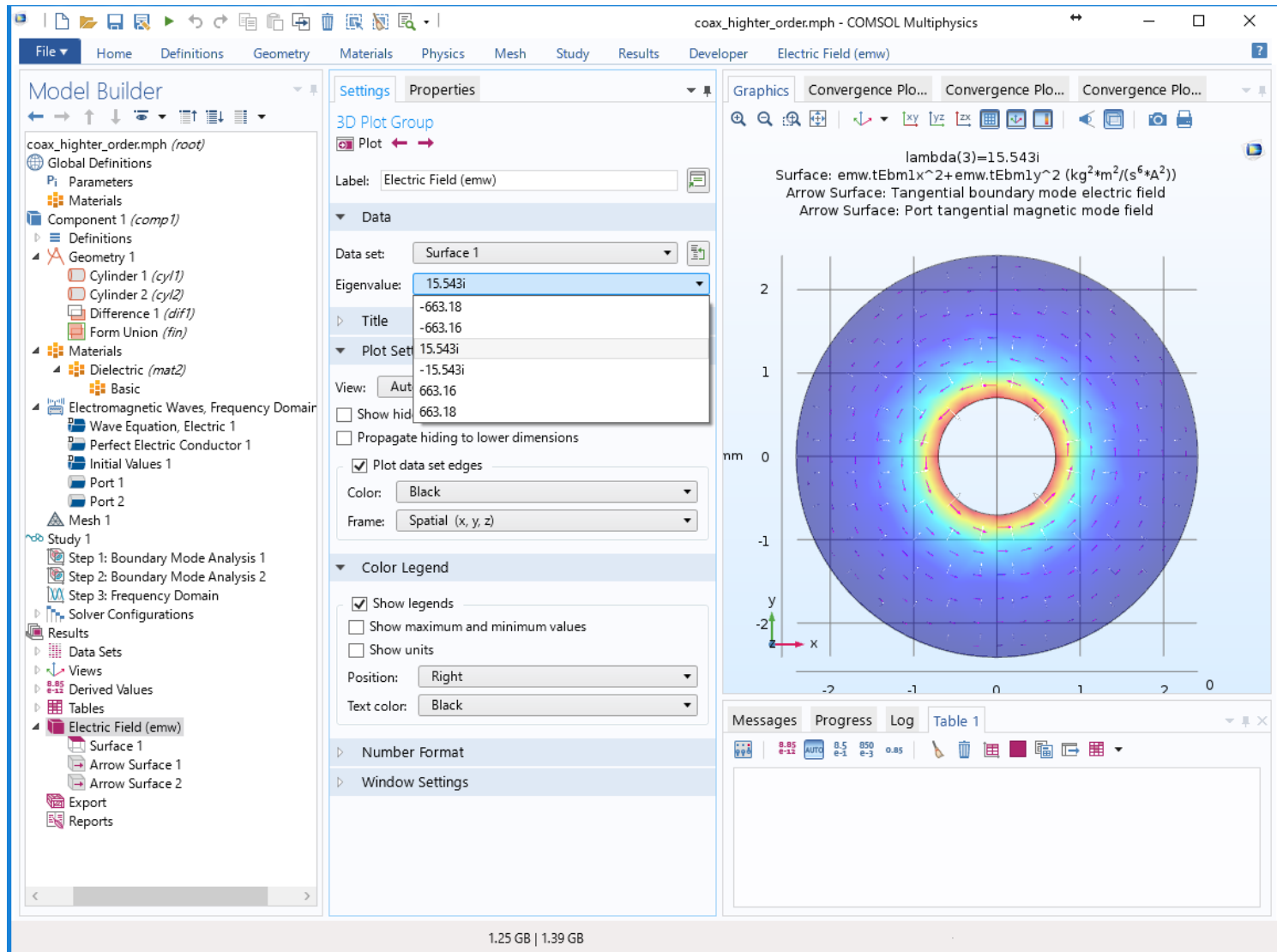
# ポートの定義

The screenshot displays the COMSOL Multiphysics interface for a model named 'coax\_higher\_order.mph'. The left-hand 'Model Builder' tree shows the hierarchy: Component 1 (comp1) > Geometry 1 > Port 1. The 'Port' settings panel is active, showing 'Label: Port 1' and 'Boundary Selection: Manual'. Under 'Boundary Selection', boundary 3 is selected and marked as 'Active'. The 'Port Properties' section is configured with 'Port name: 1', 'Type of port: Numeric', and 'Wave excitation at this port: On'. The 'Port Mode Settings' section is partially visible, showing 'Mode phase:  $\theta_{in}$  0 rad'. The right-hand 'Graphics' window shows a 3D model of a coaxial cable with a green cylindrical port at the bottom. A coordinate system (x, y, z) is shown at the bottom left of the graphics window. The bottom status bar indicates a memory usage of 1.26 GB | 1.45 GB.



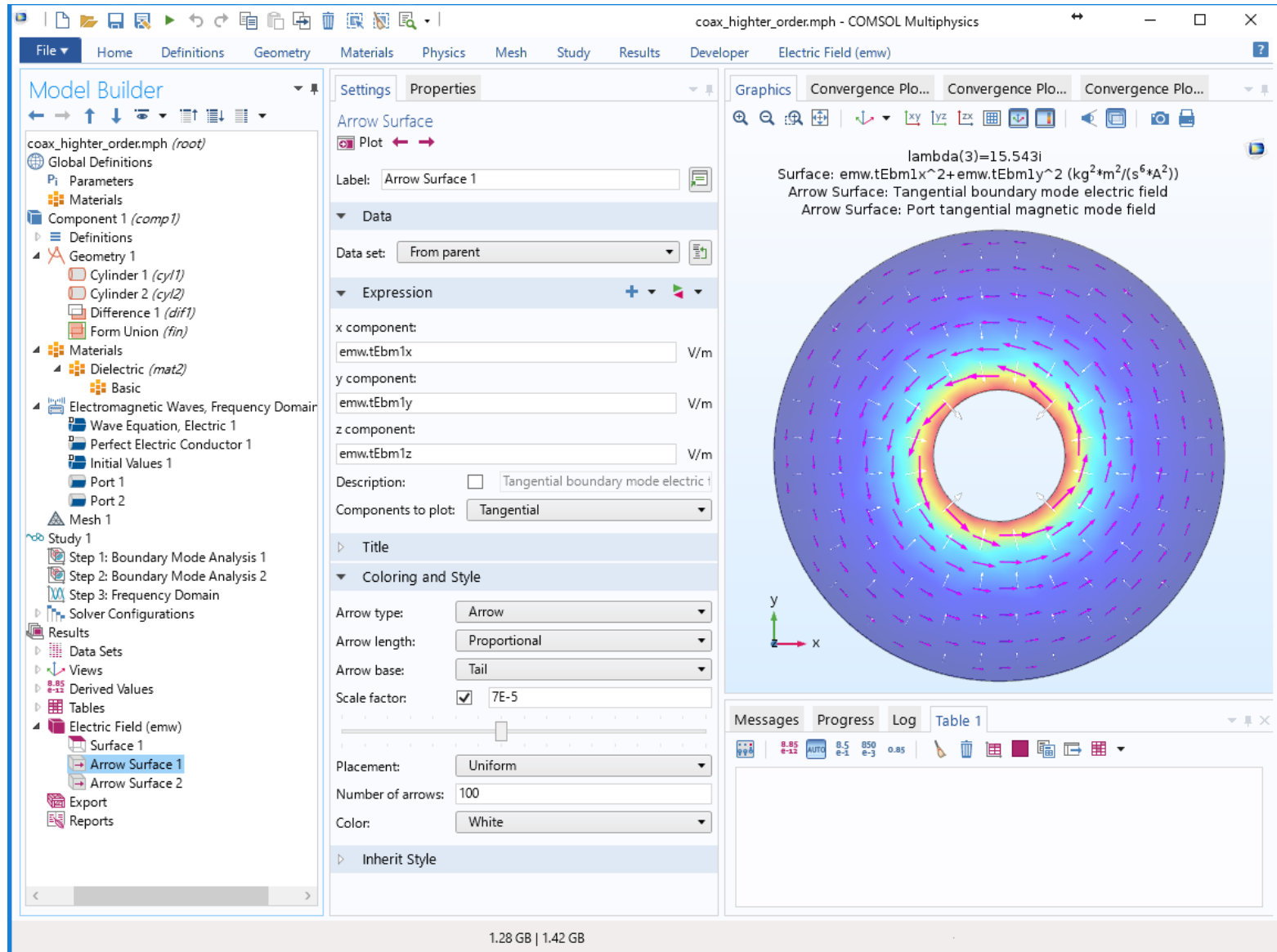


# 境界モード解析結果



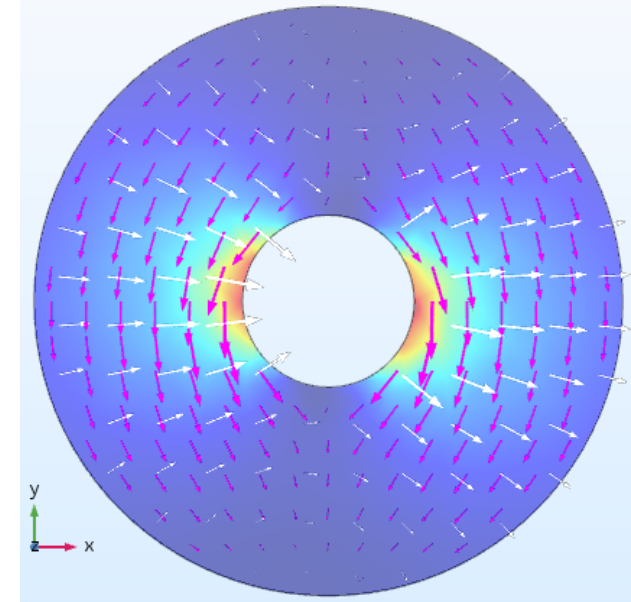
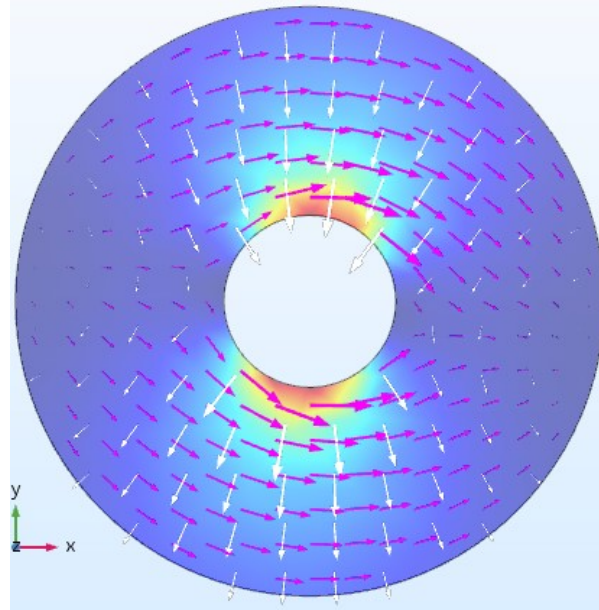
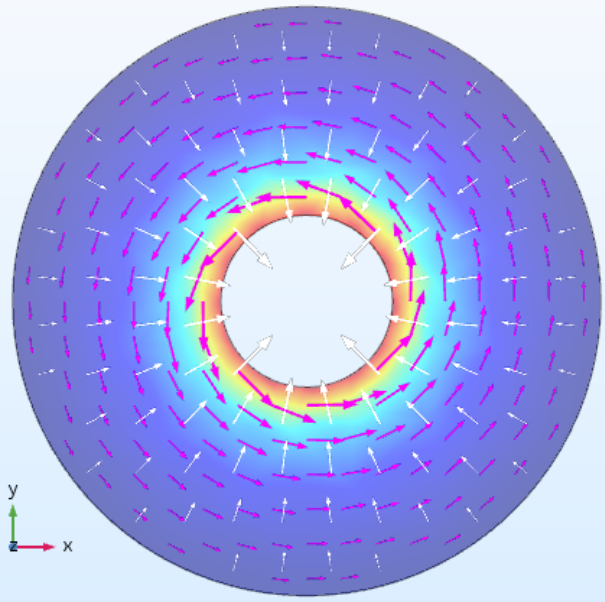


# 境界モード解析結果



## 境界モード解析結果

周波数 500 MHz



663.16

663.18

$$k_c \cong \frac{2}{a+b} j 15.5413$$

```
In[12]:= kc =  $\frac{2}{a+b}$ ;
          fc =  $\frac{kc}{2 * \pi * \sqrt{\mu * \epsilon}}$ ;
          fcGHz = fc * 10-9
```

Out[14]= 20.7538

```
In[1]:= f = 500. * 106;
        \omega = 2 * \pi * f;
        \epsilon r = 2.2;
        \epsilon 0 = 8.85418782 * 10-12;
        \mu 0 = 1.25663706 * 10-6;
        \beta = \omega * \sqrt{\mu 0 * (\epsilon r * \epsilon 0)}
```

Out[6]= 15.5432

つまり、「境界モード解析」のTransformで”None”とすると、伝搬定数 $\gamma$ が出力されている。高次モードのカットオフ周波数を調べるには、解析周波数を変えて、伝搬定数が虚数になる周波数を探せばよい。

21-22 GHzの間に高次モードカットオフ周波数があることがわかる。



# 参考) 全モードの計算と表示(1)

coax\_highter\_order.mph - COMSOL Multiphysics

File Home Definitions Geometry Materials Physics Mesh Study Results Developer

Model Builder

- coax\_highter\_order.mph (root)
  - Global Definitions
    - Parameters
    - Materials
  - Component 1 (comp1)
    - Definitions
    - Geometry 1
    - Materials
    - Electromagnetic Waves, Frequency
      - Wave Equation, Electric 1
      - Perfect Electric Conductor 1
      - Initial Values 1
      - Port 1
      - Port 2
    - Mesh 1
    - Study 1
      - Step 1: Boundary Mode Analysis 1
      - Step 2: Boundary Mode Analysis 2
      - Step 3: Frequency Domain
      - Solver Configurations
    - Results
      - Data Sets
      - Views
      - Derived Values
        - S-parameter, S11dB (emw)
        - S-parameter, S21dB (emw)
      - Tables
      - Electric Field (emw)
        - Surface 1
        - Arrow Surface 1
        - Arrow Surface 2
      - Export
      - Reports

Settings Properties

Boundary Mode Analysis

- Compute Update Solution

Label: Boundary Mode Analysis 1

Study Settings

Transform: None

Port name: 1

Mode analysis frequency: 0.5[GHz] Hz

Mode search method: All (filled matrix)

Maximum matrix size: 2000

Physics and Variables Selection

Modify physics tree and variables for study step

Physics interface	Solve	Discretization
Electromagnetic Waves, F...	<input checked="" type="checkbox"/>	Physics settings

Values of Dependent Variables

Mesh Selection

Adaptation and Error Estimates

Study Extensions

Graphics Convergence Plot 1 Convergence Plot 2

lambda(577)=10.479i

Surface:  $emw.tEbm1x^2 + emw.tEbm1y^2$  ( $kg^2 \cdot m^2 / (s^6 \cdot A^2)$ )

Arrow Surface: Tangential boundary mode electric field

Arrow Surface: Port tangential magnetic mode field

mm 0 1 2 2 1 0 mm

-2 -1 0 -1 -2

z

y x

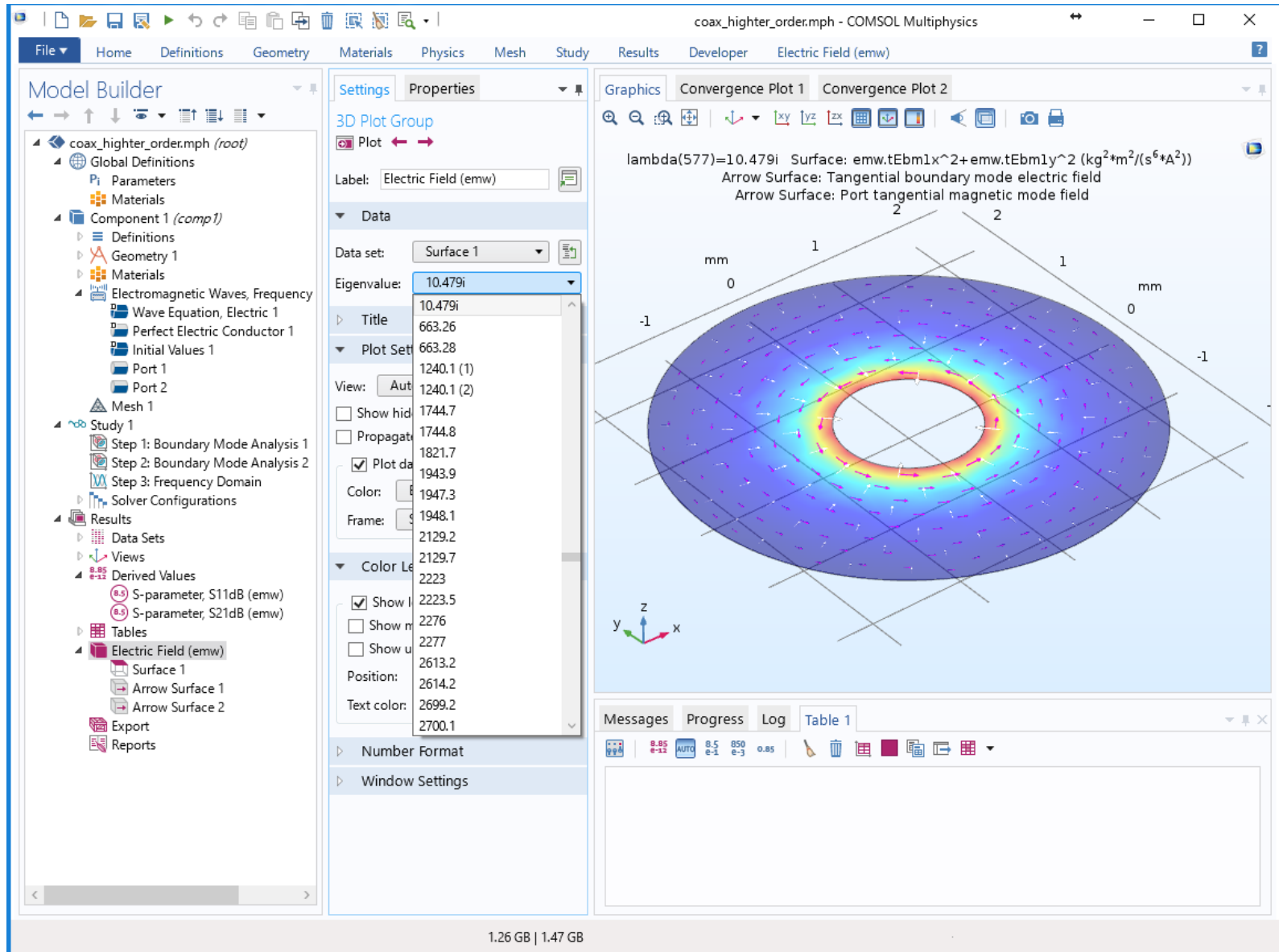
Messages Progress Log Table 1

0.85 AUTO 0.5 0.50 0.85

0.5 e-1 0.5 e-3

1.26 GB | 1.46 GB

# 参考) 全モードの計算と表示(2)



# 特性インピーダンスの解析

# 電圧計算用の経路を作成

The screenshot displays the COMSOL Multiphysics interface for a model named 'coax\_tem.mph'. The 'Model Builder' tree on the left shows the hierarchy: Global Definitions, Materials, Component 1 (comp1), Definitions, Geometry 1, Cylinder 1 (cyl1), Cylinder 2 (cyl2), Difference 1 (dif1), Line Segment 1 (ls1), Form Union (fin), Materials, Electromagnetic Waves, Frequency, Wave Equation, Electric 1, Perfect Electric Conductor 1, Initial Values 1, Port 1, Integration Line for Voltage, Integration Line for Current, Port 2, Mesh 1, Study 1, Results, Data Sets, Views, and Derived Values. The 'Derived Values' section includes S-parameter, S11dB (emw), S-parameter, S21dB (emw), and Characteristics. The 'Settings' pane for 'Line Segment 1' shows the 'Start Point' and 'Endpoint' both set to 'Vertex', with 'Start vertex' as 'dif1 15' and 'End vertex' as 'dif1 13'. The 'Selections of Resulting Entities' section is set to 'None' and 'Show in physics' is 'Edge selection'. The 'Graphics' window shows a 3D view of a coaxial cable with a grid and axes (x, y, z). The 'Messages' window at the bottom right shows a table with the following data:

freq (GHz)	TEM mode port characteristic impedance ( $\Omega$ )
0.50000	50.325





# 電圧用積分経路設定

The screenshot displays the COMSOL Multiphysics interface for a model named 'coax\_tem.mph'. The 'Model Builder' tree on the left shows the hierarchy: Component 1 (comp1) > Geometry 1 > Cylinder 1 (cyl1) > Integration Line for Voltage 1. The 'Properties' window for 'Integration Line for Voltage 1' is open, showing 'Edge Selection' with 'Manual' selection and 'Active' checked. The 'Graphics' window shows a 3D view of a coaxial cable with a blue line segment along its length, representing the integration path. The coordinate system (x, y, z) is shown at the bottom left of the graphics window, with dimensions in mm. The 'Messages' window at the bottom right shows a table with the following data:

freq (GHz)	TEM mode port characteristic impedance (Ω)
0.50000	50.325

759 MB | 992 MB

# 電流用積分経路設定

The screenshot displays the COMSOL Multiphysics interface for a model named 'coax\_tem.mph'. The 'Model Builder' tree on the left shows the hierarchy: Global Definitions, Parameters, Materials, Component 1 (comp1), Definitions, Geometry 1 (Cylinder 1, Cylinder 2, Difference 1, Line Segment 1, Form Union), Materials, Electromagnetic Waves, Frequency Domain (Wave Equation, Perfect Electric Conductor, Initial Values, Port 1, Port 2), Mesh 1, Study 1, Results (Data Sets, Views, Derived Values, Tables, Electric Field, Export, Reports). The 'Integration Line for Current 1' is selected under Port 1.

The 'Settings' pane for 'Integration Line for Current 1' shows the 'Edge Selection' method set to 'Manual'. The 'Active' edges are listed as 7, 8, 15, and 18. The 'Override and Contribution' section is currently empty.

The 'Graphics' window shows a 3D view of the coaxial cable model. A blue circle highlights the integration line on the inner conductor. The coordinate system (x, y, z) is shown at the bottom left of the graphics window, with the z-axis along the length of the cable. The dimensions are in millimeters (mm).

The 'Messages' pane at the bottom right shows a table with the following data:

freq (GHz)	TEM mode port characteristic impedance ( $\Omega$ )
0.50000	50.325

763 MB | 991 MB



# 特性インピーダンスの計算

The screenshot displays the COMSOL Multiphysics interface for a simulation titled 'coax\_tem.mph'. The left-hand 'Model Builder' pane shows a hierarchical tree of the model components, including 'Component 1 (comp1)' with 'Geometry 1' (two cylinders and a difference) and 'Materials' (Perfect Electric Conductor 1). The 'Study 1' section includes 'Results' with 'Derived Values' containing 'Characteristics'. The central 'Settings' pane is set to 'Properties' for 'Global Evaluation', with 'Label' set to 'Characteristics' and 'Data set' as 'Study 1/Solution 1 (sol1)'. The 'Expressions' table lists 'emw.Zmode\_1' with unit  $\Omega$  and description 'TEM mode port character...'. The right-hand 'Graphics' pane shows a 3D wireframe model of a coaxial cable with a length of 5 mm and a diameter of 2 mm. The bottom status bar shows '789 MB | 996 MB'. A red box highlights a table in the 'Table 2' window with the following data:

freq (GHz)	TEM mode port characteristic impedance ( $\Omega$ )
0.50000	50.325