

JPEGのアルゴリズムについて

JPEGアルゴリズムの確認

[Reference]

<http://en.wikipedia.org/wiki/Jpeg>

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BMPファイルの読み込み

```
In[1]:= SetDirectory["d:/hira2/public_html/hobby/edu/jpeg"]
```

```
Out[1]= d:\hira2\public_html\hobby\edu\jpeg
```

```
SetDirectory["d:/Home/takuichi/hira2/public_html/hobby/edu/jpeg"]
```

```
In[2]:= g = Import["a.bmp"]
```

```
Out[2]= - Graphics -
```

```
In[3]:= Show[g]
```

```
A
```

```
Out[3]= - Graphics -
```

InputForm[g]

```
In[4]:= ImgList = g[[1, 1]]; (* {R,G,B} の2次元リスト *)
```

```
ImgListRed = Map[#[[1]] &, ImgList, {2}]; (* R (0-255) の2次元リスト *)
```

```
ImgListGreen = Map[#[[2]] &, ImgList, {2}]; (* G (0-255) の2次元リスト *)
```

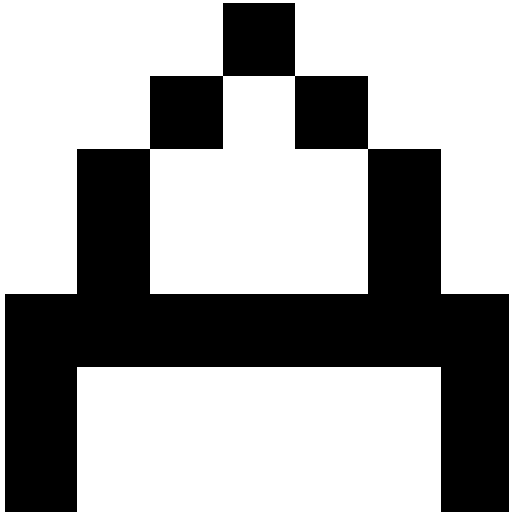
```
ImgListBlue = Map[#[[3]] &, ImgList, {2}]; (* B (0-255) の2次元リスト *)
```

```
nx = Length[ImgList[[1]]];
```

```
ny = Length[ImgList];
```

```
In[9]:= MakeRGBRasterImage[ImgListRed_, ImgListGreen_, ImgListBlue_] := Module[{},
Graphics[Raster[Table[{ImgListRed[[j, i]],
ImgListGreen[[j, i]], ImgListBlue[[j, i]]}, {j, 1, ny}, {i, 1, nx}],
{{0, 0}, {nx, ny}}, {0, 255}, ColorFunction->RGBColor]
];
```

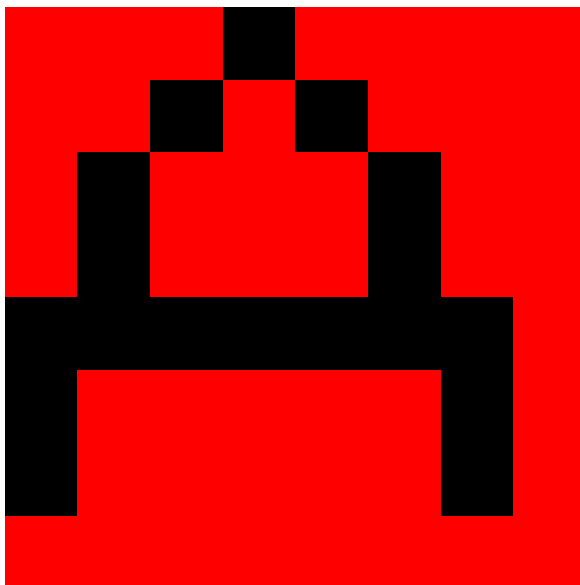
```
In[10]:= Show[MakeRGBRasterImage[ImgListRed, ImgListGreen, ImgListBlue],  
            AspectRatio -> Automatic]
```



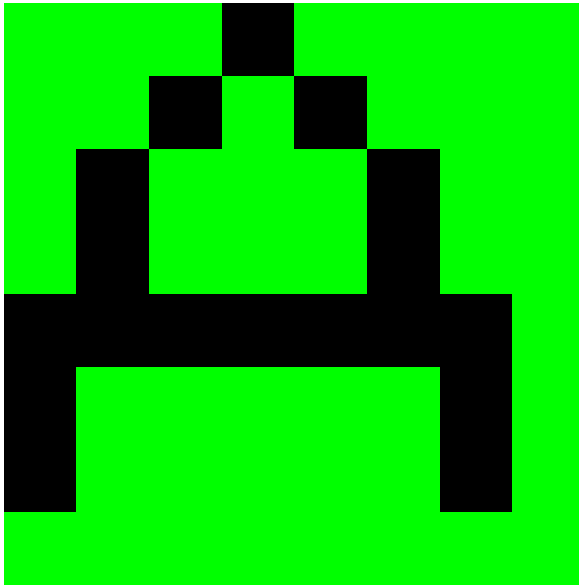
```
Out[10]= - Graphics -
```

```
In[11]:= ZeroTable = Table[0, {n, 0, ny - 1}, {m, 0, nx - 1}];  
Print["**** Red Component ****"];  
Show[MakeRGBRasterImage[ImgListRed, ZeroTable, ZeroTable],  
      AspectRatio -> Automatic];  
Print["**** Green Component ****"];  
Show[MakeRGBRasterImage[ZeroTable, ImgListGreen, ZeroTable],  
      AspectRatio -> Automatic];  
Print["**** Blue Component ****"];  
Show[MakeRGBRasterImage[ZeroTable, ZeroTable, ImgListBlue],  
      AspectRatio -> Automatic];
```

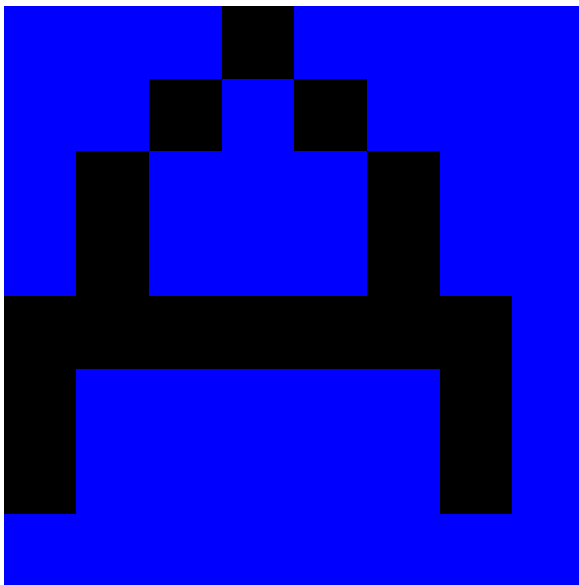
```
**** Red Component ****
```



**** Green Component ****



**** Blue Component ****



$(R,G,B) \rightarrow (Y, Cb, Cr)$

JPEGではRGBでなく、YCbCr(またはYUVとも言われる)でそれぞれの成分を処理する。YUVはテレビ放送のPAL方式で採用されている色表現。

<http://en.wikipedia.org/wiki/YCbCr>

The Y component represents the brightness of a pixel

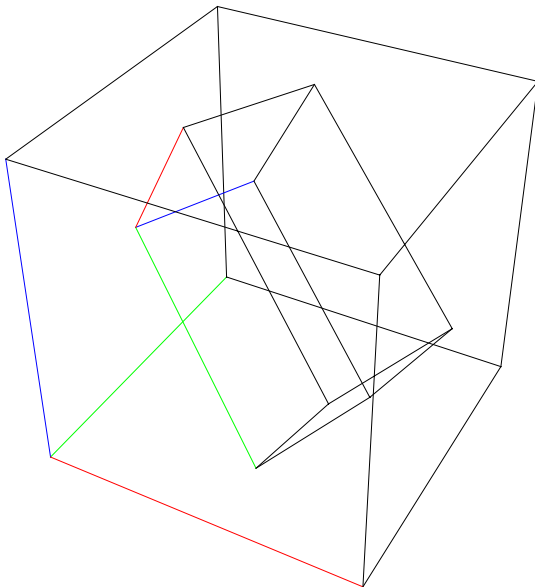
The Cb and Cr components together represent the chrominance

The human eye can see more detail in the Y component than in Cb and

Cr. Using this knowledge, encoders can be designed to compress images more efficiently.

```
In[18]:= YCbCrFromRGB[{r_, g_, b_}] := {0.299 * r + 0.587 * g + 0.114 * b,
      128 - 0.168736 * r - 0.331264 * g + 0.5 * b,
      128 + 0.5 * r - 0.418688 * g - 0.081312 * b};
RGBFromYCbCr[{y_, cb_, cr_}] :=
  {-179.45579155073017` - 1.2175717664097974` * cb + 1.4019995886573404` * cr + y,
   135.45879483222961` - 0.3441356788389385` * cb - 0.7141361555818125` * cr + y,
   -226.81606046360054` + 1.7720000660738162` * cb + 4.062980628562637` * cr + y};

In[20]:= p[0] = {0, 0, 0};
p[1] = {255, 0, 0};
p[2] = {255, 255, 0};
p[3] = {0, 255, 0};
p[4] = {0, 0, 255};
p[5] = {255, 0, 255};
p[6] = {255, 255, 255};
p[7] = {0, 255, 255};
pcubeRGB = {{p[0], p[1]}, {p[1], p[2]}, {p[2], p[3]}, {p[3], p[0]},
  {p[4], p[5]}, {p[5], p[6]}, {p[6], p[7]}, {p[7], p[4]},
  {p[0], p[4]}, {p[1], p[5]}, {p[2], p[6]}, {p[3], p[7]}};
pcubeYCbCr = Map[YCbCrFromRGB, pcubeRGB, {2}];
pcubeRGB2 = {{RGBColor[1, 0, 0], Line[{p[0], p[1]}]},
  {RGBColor[0, 1, 0], Line[{p[0], p[3]}]}, {RGBColor[0, 0, 1], Line[{p[0], p[4]}]}};
pcubeYCbCr2 = {{RGBColor[1, 0, 0], Line[{YCbCrFromRGB[p[0]], YCbCrFromRGB[p[1]]]}]},
  {RGBColor[0, 1, 0], Line[{YCbCrFromRGB[p[0]], YCbCrFromRGB[p[3]]]}]},
  {RGBColor[0, 0, 1], Line[{YCbCrFromRGB[p[0]], YCbCrFromRGB[p[4]]]}]}};
Show[Graphics3D[Map[Line, pcubeRGB, 1], pcubeRGB2],
  Graphics3D[Map[Line, pcubeYCbCr, 1], pcubeYCbCr2]],
  Boxed -> False];
```

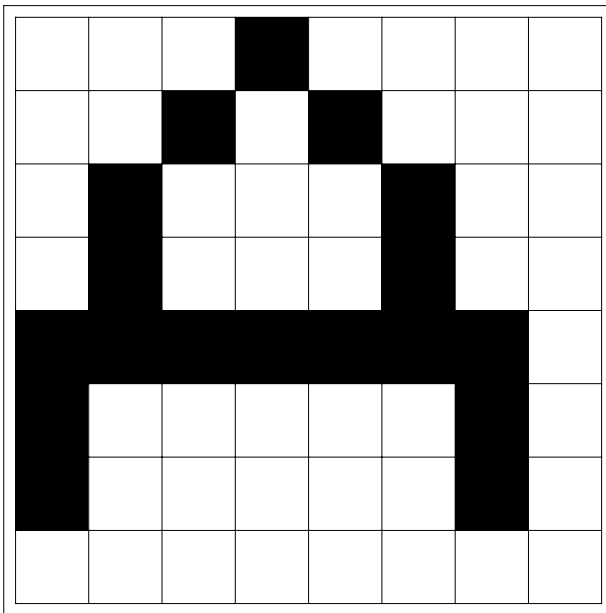


```
In[34]:= ImgListY = Map[(YCbCrFromRGB[#][[1]] &), ImgList, {2}]; (* Yの2次元リスト *)
ImgListCb = Map[(YCbCrFromRGB[#][[2]] &), ImgList, {2}]; (* Cbの2次元リスト *)
ImgListCr = Map[(YCbCrFromRGB[#][[3]] &), ImgList, {2}]; (* Crの2次元リスト *)

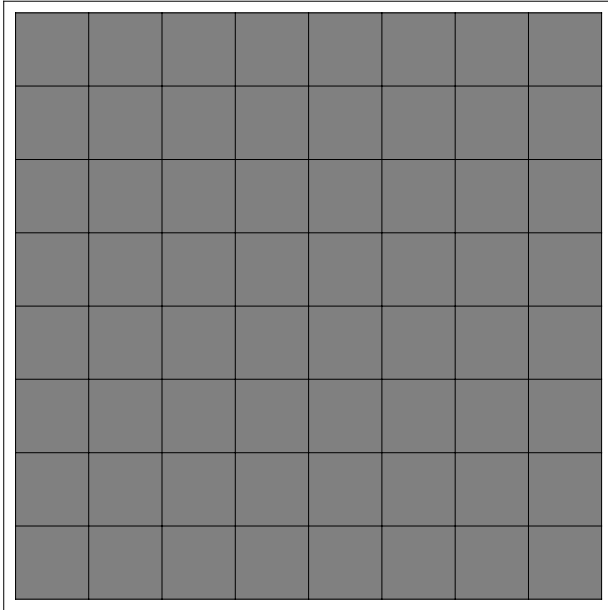
ImgListY = ImgListY - 128; (* Yは128を引いて-128-127の範囲にする *)
```

```
In[35]:= Print["**** Y Component ****"];
ListDensityPlot[ImgListY,
  FrameTicks → False,
  PlotRange → {-128, 127}];
Print["**** Cb Component ****"];
ListDensityPlot[ImgListCb,
  FrameTicks → False,
  PlotRange → {0, 255}];
Print["**** Cr Component ****"];
ListDensityPlot[ImgListCr,
  FrameTicks → False,
  PlotRange → {0, 255}];
```

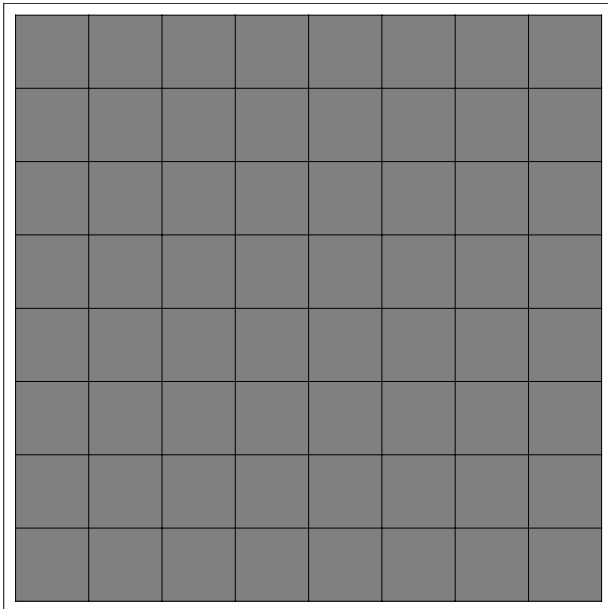
**** Y Component ****



**** Cb Component ****



**** Cr Component ****



DFT (Discrete Fourier Transform)

JPEGではDCTが使われるので、これは関係ないが、基本的なDFTと比較するために計算してる。

```
In[44]:= FImgListY = Fourier[ImgListY];  
FImgListCb = Fourier[ImgListCb];  
FImgListCr = Fourier[ImgListCr];
```

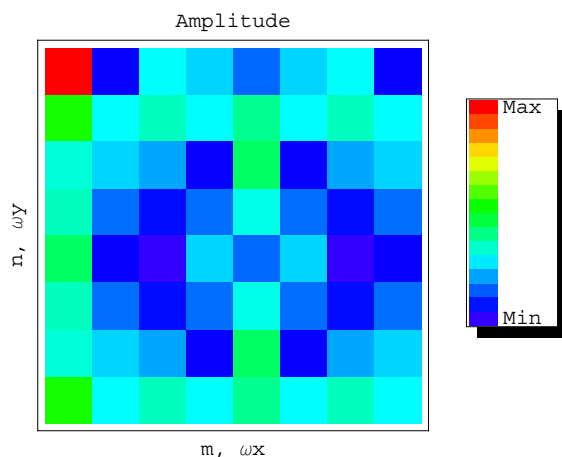
```

In[47] := << Graphics`Legend`;
ShowLegend[
  ListDensityPlot[Abs[ Reverse[FImgListY] ],
    Mesh → False,
    PlotRange → Automatic,
    PlotLabel → "Amplitude",
    FrameLabel → {"m, ωx", "n, ωy"},
    FrameTicks → None,
    ColorFunction → (Hue[-0.7 * (# - 1), 1, 1] &),
    DisplayFunction → Identity]
  , {(Hue[0.7 * (#), 1, 1] &), 16, "Max", "Min",
    LegendPosition → {1.1, -.4}, LegendSize → {0.4, 1.}, LegendLabel → ""}
];

colfun[x_] := RGBColor[0, 3 * x, 1] /; (x ≤ 1 / 3);
colfun[x_] := RGBColor[3 * (x - 1 / 3), 1, 1] /; (1 / 3 < x ≤ 2 / 3);
colfun[x_] := RGBColor[-3 * (x - 2 / 3) + 1, -3 * (x - 2 / 3) + 1, 1] /; (2 / 3 < x);

ShowLegend[
  ListDensityPlot[Arg[Reverse[FImgListY]] *  $\frac{180}{\pi}$ ,
    Mesh → False,
    PlotRange → {-180, 180},
    PlotLabel → "Phase",
    FrameLabel → {"m, ωx", "n, ωy"},
    FrameTicks → None,
    ColorFunction → colfun,
    DisplayFunction → Identity]
  , {colfun[-# + 1] &, 16, "180", "-180",
    LegendPosition → {1.1, -.4}, LegendSize → {0.4, 1.}, LegendLabel → "deg"}
];

```

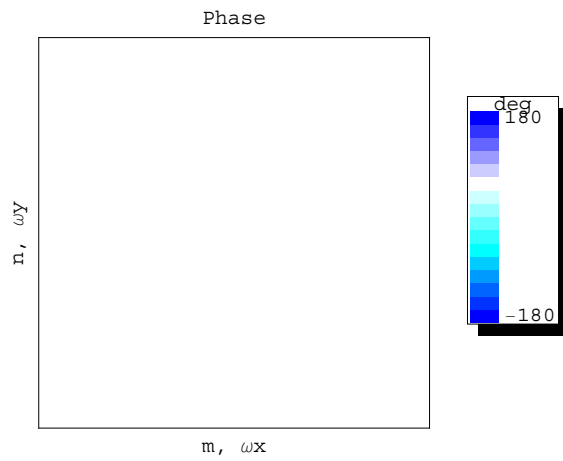


Arg::indet : 不定式Arg[0. + 0. i]が見つかりました . [詳細](#)

Arg::indet : 不定式Arg[0. + 0. i]が見つかりました . [詳細](#)

DensityGraphics::zval : 非数値Interval[{-180, 180}]が第1引数の位置{4, 3}で見つかりました . [詳細](#)

DensityGraphics::zval : 非数値Interval[{-180, 180}]が第1引数の位置{4, 7}で見つかりました . [詳細](#)



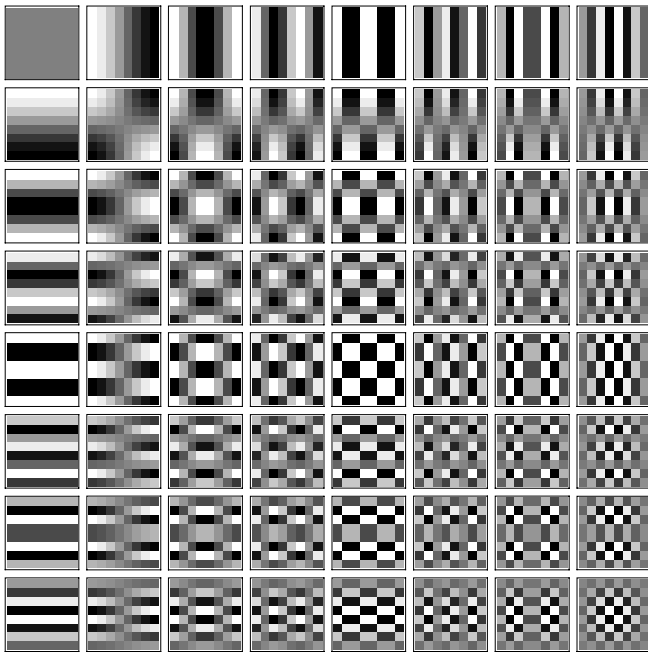
DCT (Discrete Cosine Transform; 離散コサイン変換)

基底の描画 (計算と無関係。教育用)


```

In[53]:= c[p_] := If[p == 0, 1/√2, 1];
b[u_, v_, m_, n_] :=  $\frac{2}{nn} * c[u] * c[v] * \text{Cos}\left[\frac{(2 * m + 1) * u * \pi}{2 * nn}\right] * \text{Cos}\left[\frac{(2 * n + 1) * v * \pi}{2 * nn}\right]$ ;
nn = 8;
buv[u_, v_] :=
  ListDensityPlot[Reverse[Table[b[u, v, m, n], {m, 0, nn - 1}, {n, 0, nn - 1}]],
    Mesh → False,
    Frame → True,
    FrameTicks → None,
    DisplayFunction → Identity];
Show[GraphicsArray[Table[buv[u, v], {u, 0, nn - 1}, {v, 0, nn - 1}]],
  ImageSize → {320, 320}]

```



Out[57]= - GraphicsArray -

2次元リストにDCTを行う関数の定義

[参考]

貴家仁志：よくわかるデジタル画像処理、CQ出版社、p.123

```

In[58]:= DCT[lis_] := Module[{c, g, nn},
  c[p_] := If[p == 0, 1/√2, 1];
  nn = Length[lis];
  g[u_, v_] :=  $\frac{2}{nn} * c[u] * c[v] * \text{Sum}[lis[[m + 1, n + 1]] * \text{Cos}\left[\frac{(2 * m + 1) * u * \pi}{2 * nn}\right] * \text{Cos}\left[\frac{(2 * n + 1) * v * \pi}{2 * nn}\right], \{m, 0, nn - 1\}, \{n, 0, nn - 1\}]$ ;
  Table[g[u, v], {u, 0, nn - 1}, {v, 0, nn - 1}];
];

```

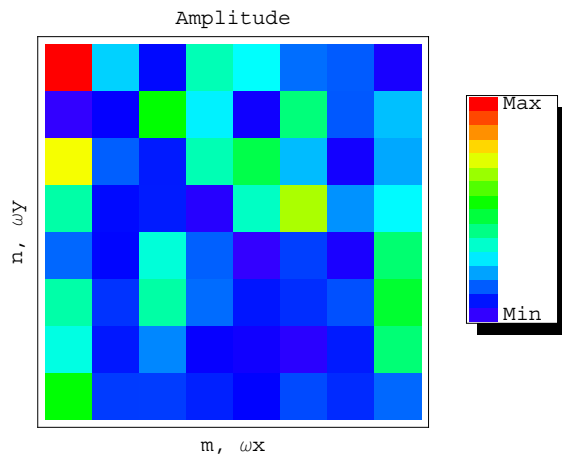


```

In[65]:= << Graphics`Legend`;
ShowLegend[
  ListDensityPlot[Abs[ Reverse[FImgListY] ],
    Mesh → False,
    PlotRange → Automatic,
    PlotLabel → "Amplitude",
    FrameLabel → {"m, ωx", "n, ωy"},
    FrameTicks → None,
    ColorFunction → (Hue[-0.7 * (# - 1), 1, 1] &),
    DisplayFunction → Identity]
, {(Hue[0.7 * (#), 1, 1] &), 16, "Max", "Min",
  LegendPosition → {1.1, -.4}, LegendSize → {0.4, 1.}, LegendLabel → ""}
];

colfun[x_] := RGBColor[0, 3 * x, 1] /; (x ≤ 1 / 3);
colfun[x_] := RGBColor[3 * (x - 1 / 3), 1, 1] /; (1 / 3 < x ≤ 2 / 3);
colfun[x_] := RGBColor[-3 * (x - 2 / 3) + 1, -3 * (x - 2 / 3) + 1, 1] /; (2 / 3 < x);

```



Quantization (量子化)

量子化テーブル

```
In[70]:= qty = 
$$\begin{pmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{pmatrix};$$

```

(* Y成分用 *)

```
qtcbcr = 
$$\begin{pmatrix} 17 & 18 & 24 & 47 & 99 & 99 & 99 & 99 \\ 18 & 21 & 26 & 66 & 99 & 99 & 99 & 99 \\ 24 & 26 & 56 & 99 & 99 & 99 & 99 & 99 \\ 47 & 66 & 99 & 99 & 99 & 99 & 99 & 99 \\ 99 & 99 & 99 & 99 & 99 & 99 & 99 & 99 \\ 99 & 99 & 99 & 99 & 99 & 99 & 99 & 99 \\ 99 & 99 & 99 & 99 & 99 & 99 & 99 & 99 \\ 99 & 99 & 99 & 99 & 99 & 99 & 99 & 99 \end{pmatrix};$$

```

(* Cb,Cr成分用 *)

```
In[76]:= text = Table[Text[StyleForm[
ToString[Transpose[Reverse[qty]][[i, j]]], FontSize -> 16, FontWeight -> "Bold",
FontColor -> Hue[0.7 * (Transpose[Reverse[qty]][[i, j]] / 255.), 1, 1]],
{i, j}], {i, 1, 8}, {j, 1, 8}];
Show[Graphics[{text}],
PlotRange -> {{0.5, 8.5}, {0.5, 8.5}},
AspectRatio -> Automatic];
```

```
16  11  10  16  24  40  51  61
12  12  14  19  26  58  60  55
14  13  16  24  40  57  69  56
14  17  22  29  51  87  80  62
18  22  37  56  68 109 103  77
24  35  55  64  81 104 113  92
49  64  78  87 103 121 120 101
72  92  95  98 112 100 103  99
```

量子化する
sは品質 (大きいほど良い)

```

In[85]:= s = 1;

FImgListY2 = Round[FImgListY / (qty / s)];
FImgListCb2 = Round[FImgListCb / (qtcbcr / s)];
FImgListCr2 = Round[FImgListCr / (qtcbcr / s)];

Print[FImgListY2 // MatrixForm];
Print[FImgListCb2 // MatrixForm];
Print[FImgListCr2 // MatrixForm];


$$\begin{pmatrix} 28 & -10 & 2 & -10 & 5 & -2 & 1 & 0 \\ 0 & 2 & -17 & -6 & -1 & -3 & 1 & -2 \\ 24 & 5 & 2 & 7 & -5 & -2 & 0 & -2 \\ 12 & 1 & 1 & 0 & 3 & 3 & -1 & 2 \\ -4 & -1 & 4 & 1 & 0 & 0 & 0 & 2 \\ -7 & -1 & 3 & 1 & 0 & 0 & 0 & -2 \\ 3 & 0 & -1 & 0 & 0 & 0 & 0 & 2 \\ 3 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{pmatrix}$$



$$\begin{pmatrix} 60 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$



$$\begin{pmatrix} 60 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$


```

本来ならばここでジグザグスキャンを行った後、エントロピー符号化（ハフマン符号化または算術符号化）が入り、JPEGファイルが生成される。

```

In[92]:= dx = dy = 1.;
ggrid = Table[{Line[{{dx*(i-1), 0}, {dx*(i-1), dy*8}],
  Line[{{0, dy*(j-1)}, {dx*8, dy*(j-1)}}]}, {i, 1, 9}, {j, 1, 9}];

zigzagmat = 
$$\begin{pmatrix} 1 & 2 & 6 & 7 & 15 & 16 & 28 & 29 \\ 3 & 5 & 8 & 14 & 17 & 27 & 30 & 43 \\ 4 & 9 & 13 & 18 & 26 & 31 & 42 & 44 \\ 10 & 12 & 19 & 25 & 32 & 41 & 45 & 54 \\ 11 & 20 & 24 & 33 & 40 & 46 & 53 & 55 \\ 21 & 23 & 34 & 39 & 47 & 52 & 56 & 61 \\ 22 & 35 & 38 & 48 & 51 & 57 & 60 & 62 \\ 36 & 37 & 49 & 50 & 58 & 59 & 63 & 64 \end{pmatrix};$$


mat = Transpose[Reverse[zigzagmat]];
zigzaglin = Line[Table[Position[mat, i][[1]] - {dx/2, dy/2}, {i, 1, 64}]];
text =
  Table[{Text[StyleForm[ToString[mat[[i, j]]], FontSize -> 16, FontWeight -> "Bold"],
    {i - dx/2, j - dy/2}], {i, 1, 8}, {j, 1, 8}];
Show[Graphics[{ggrid, {RGBColor[1, 0, 1], AbsoluteThickness[2], zigzaglin}, text}],
  AspectRatio -> Automatic];

```

1	2	6	7	15	16	28	29
3	5	8	14	17	27	30	43
4	9	13	18	26	31	42	44
10	12	19	25	32	41	45	54
11	20	24	33	40	46	53	55
21	23	34	39	47	52	56	61
22	35	38	48	51	57	60	62
36	37	49	50	58	59	63	64

ここから先はJPEGファイルの展開過程である。

Inverse Quantization (逆量子化)

```
In[105] :=
  FImgListY3 = Round[FImgListY2 * (qty / s)];
  FImgListCb3 = Round[FImgListCb2 * (qtcbcr / s)];
  FImgListCr3 = Round[FImgListCr2 * (qtcbcr / s)];

  Print[FImgListY3 // MatrixForm];
  Print[FImgListCb3 // MatrixForm];
  Print[FImgListCr3 // MatrixForm];
```

$$\begin{pmatrix} 448 & -110 & 20 & -160 & 120 & -80 & 51 & 0 \\ 0 & 24 & -238 & -114 & -26 & -174 & 60 & -110 \\ 336 & 65 & 32 & 168 & -200 & -114 & 0 & -112 \\ 168 & 17 & 22 & 0 & 153 & 261 & -80 & 124 \\ -72 & -22 & 148 & 56 & 0 & 0 & 0 & 154 \\ -168 & -35 & 165 & 64 & 0 & 0 & 0 & -184 \\ 147 & 0 & -78 & 0 & 0 & 0 & 0 & 202 \\ 216 & 0 & 0 & 0 & 0 & 100 & 0 & -99 \end{pmatrix}$$

$$\begin{pmatrix} 1020 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1020 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

IDCT (Inverse Discrete Cosine Transform; 逆離散コサイン変換)

```
In[112] :=
  IDCT[lis_] := Module[{c, f, nn},
    c[p_] := If[p == 0, 1/√2, 1];
    nn = Length[lis];
    f[m_, n_] :=  $\frac{2}{nn} * \text{Sum}[c[u] * c[v] * lis[[u+1, v+1]] * \text{Cos}\left[\frac{(2 * m + 1) * u * \pi}{2 * nn}\right] * \text{Cos}\left[\frac{(2 * n + 1) * v * \pi}{2 * nn}\right], \{u, 0, nn - 1\}, \{v, 0, nn - 1\}];$ 
    Table[f[u, v], {u, 0, nn - 1}, {v, 0, nn - 1}];
  ];
```

```
In[116] :=
  IFFImgListY = IDCT[FImgListY3];
  IFFImgListCb = IDCT[FImgListCb3];
  IFFImgListCr = IDCT[FImgListCr3];
```

(Y, Cb, Cr)→(R,G,B)

(Y, Cb, Cr)→(R,G,B)の式の導出 (教育用)

```
In[119]:=
Clear[y, cb, cr, r, g, b];
Solve[{y == 0.299 * r + 0.587 * g + 0.114 * b,
       cb == 128 - 0.168736 * r - 0.331264 * g + 0.5 * b,
       cr == 128 + 0.5 * r - 0.418688 * g - 0.081312 * b}, {r, g, b}] // Simplify
```

```
Out[120]=
{{r → -179.456 - 1.21889 × 10-6 cb + 1.402 cr + 1. y,
  g → 135.459 - 0.344136 cb - 0.714136 cr + 1. y,
  b → -226.816 + 1.772 cb + 4.06298 × 10-7 cr + 1. y}}
```

(Y, Cb, Cr)→(R,G,B)

を計算

```
In[121]:=
IFFImgListY = IFFImgListY + 128; (* Yは128を足して0-255の範囲にする *)
ImgList2 = Table[{IFFImgListY[[i, j]],
                 IFFImgListCb[[i, j]], IFFImgListCr[[i, j]]}, {i, 1, nx}, {j, 1, ny}];
```

```
In[124]:=
ImgListR2 = Map[(RGBFromYCbCr[#][[1]] &), ImgList2, {2}] // Round;
(* Rの2次元リスト *)
ImgListG2 = Map[(RGBFromYCbCr[#][[2]] &), ImgList2, {2}] // Round;
(* Gの2次元リスト *)
ImgListB2 = Map[(RGBFromYCbCr[#][[3]] &), ImgList2, {2}] // Round;
(* Bの2次元リスト *)
```


画像を表示

```
In[125]:= Show[MakeRGBRasterImage[ImgListR2, ImgListG2, ImgListB2],  
  AspectRatio -> Automatic];
```

