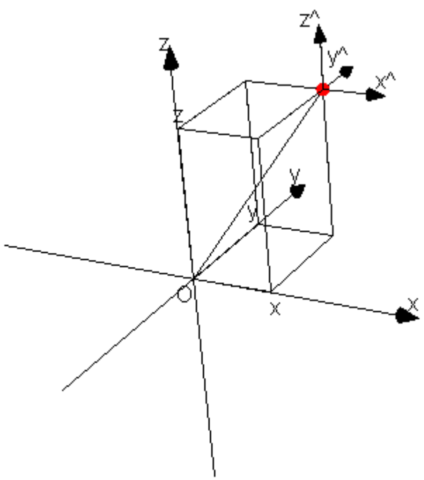
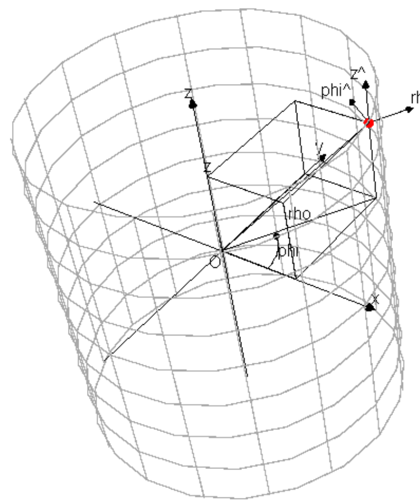
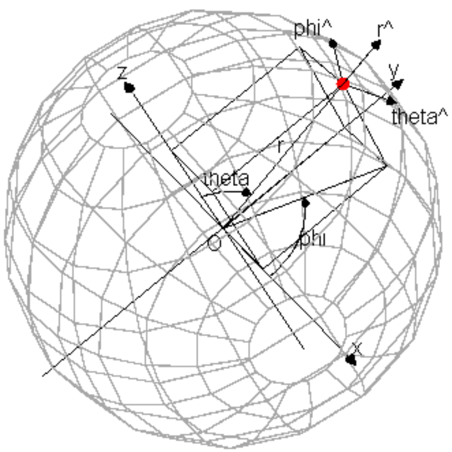
いろいろな座標系(Coordinate Systems)



Cartesian



Cylindrical



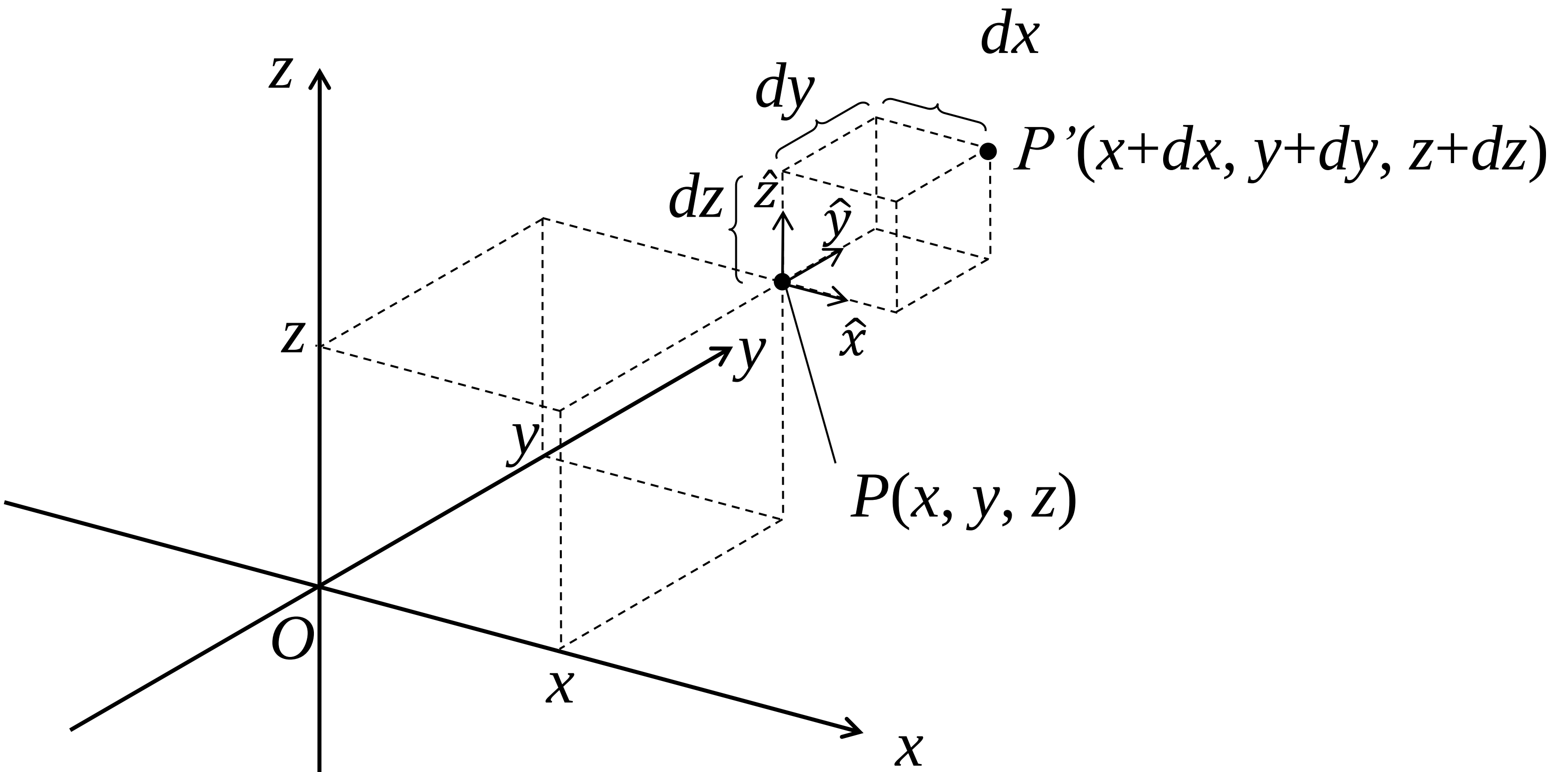
Spherical

平野拓一

座標系(Coordinate Systems)

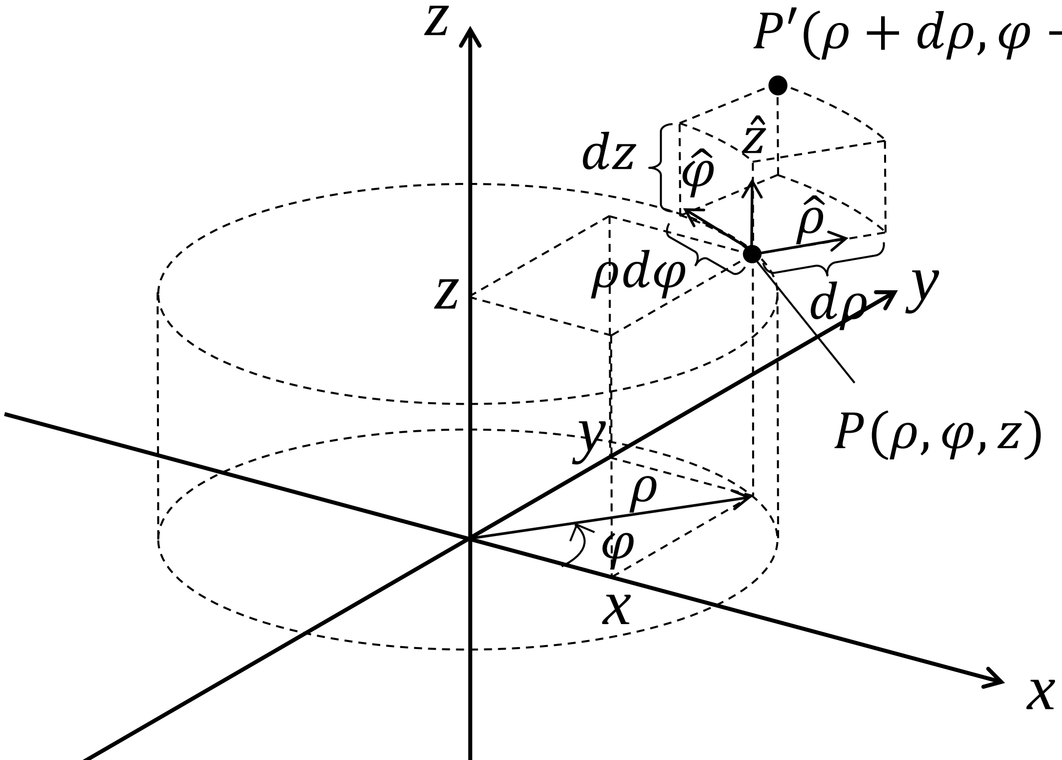
- 直交座標系(Cartesian Coordinates)
- 円筒座標系(Cylindrical Coordinates)
- 球座標系(Spherical Coordinates)

直交座標系(Cartesian Coordinates)

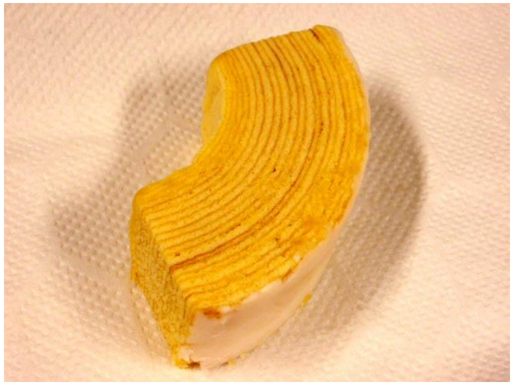


変数(Variables)	x, y, z
基底(Bases)	$\hat{x}, \hat{y}, \hat{z}$
線素(Line element)	$d\boldsymbol{l} = \overrightarrow{PP'} = \hat{x}dx + \hat{y}dy + \hat{z}dz$
面素(Surface element)	$d\boldsymbol{S}_x = \hat{x}dydz, d\boldsymbol{S}_y = \hat{y}dxdz, d\boldsymbol{S}_z = \hat{z}dxdy$
体積素(Volume element)	$dV = dxdydz$

円筒座標系(Cylindrical Coordinates)

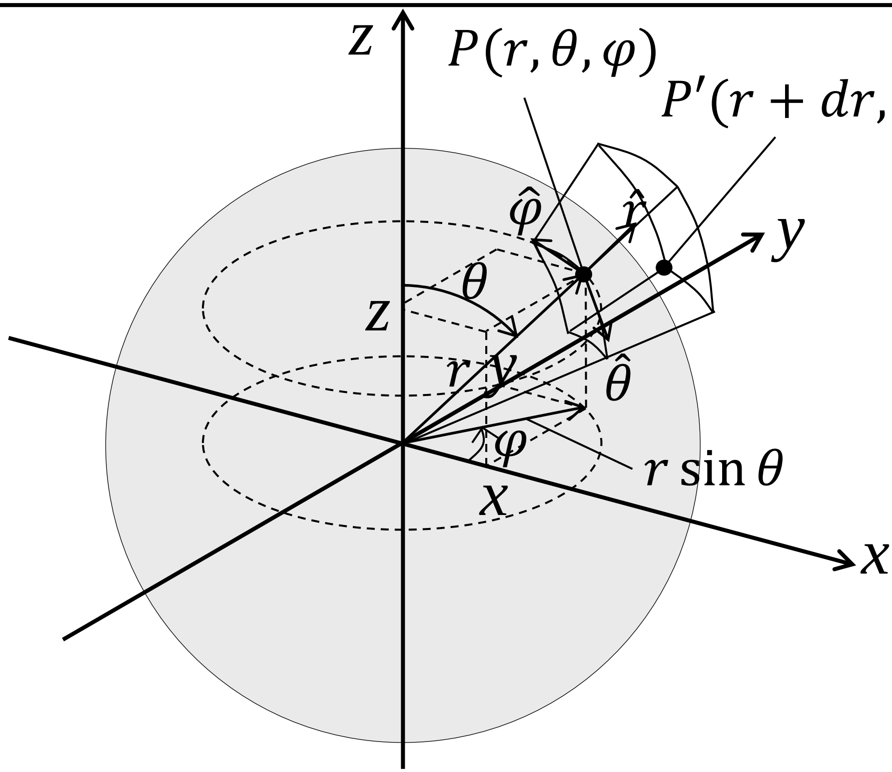


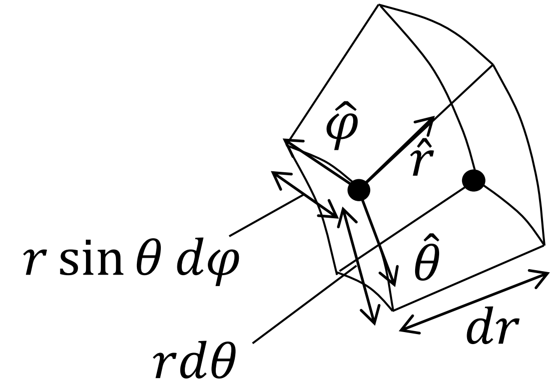
Baumkuchen

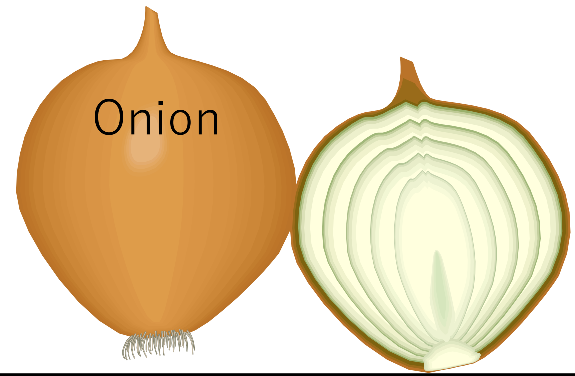


変数(Variables)	ρ, φ, z
基底(Bases)	$\hat{\rho}, \hat{\varphi}, \hat{z}$
線素(Line element)	$d\boldsymbol{l} = \overrightarrow{PP'} = \hat{\rho} d\rho + \hat{\varphi} \rho d\varphi + \hat{z} dz$
面素(Surface element)	$d\boldsymbol{S}_\rho = \hat{\rho} \rho d\varphi dz, d\boldsymbol{S}_\varphi = \hat{\varphi} d\rho dz, d\boldsymbol{S}_z = \hat{z} \rho d\rho d\varphi$
体積素(Volume element)	$dV = \rho d\rho d\varphi dz$

球座標系(Spherical Coordinates)







変数(Variables)	r, θ, φ
基底(Bases)	$\hat{r}, \hat{\theta}, \hat{\varphi}$
線素(Line element)	$d\boldsymbol{l} = \overrightarrow{PP'} = \hat{r}dr + \hat{\theta}rd\theta + \hat{\varphi}r \sin \theta \, d\varphi$
面素(Surface element)	$d\boldsymbol{S}_r = \hat{r}r^2 \sin \theta \, d\theta d\varphi, d\boldsymbol{S}_\theta = \hat{\varphi}r \sin \theta \, drd\varphi, d\boldsymbol{S}_\varphi = \hat{z}rdrd\theta$
体積素(Volume element)	$dV = r^2 \sin \theta \, drd\theta d\varphi$

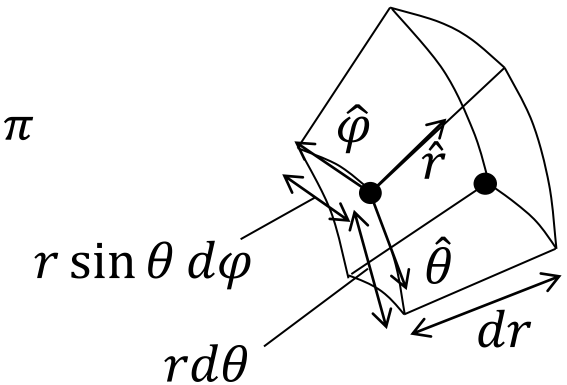
球の表面積

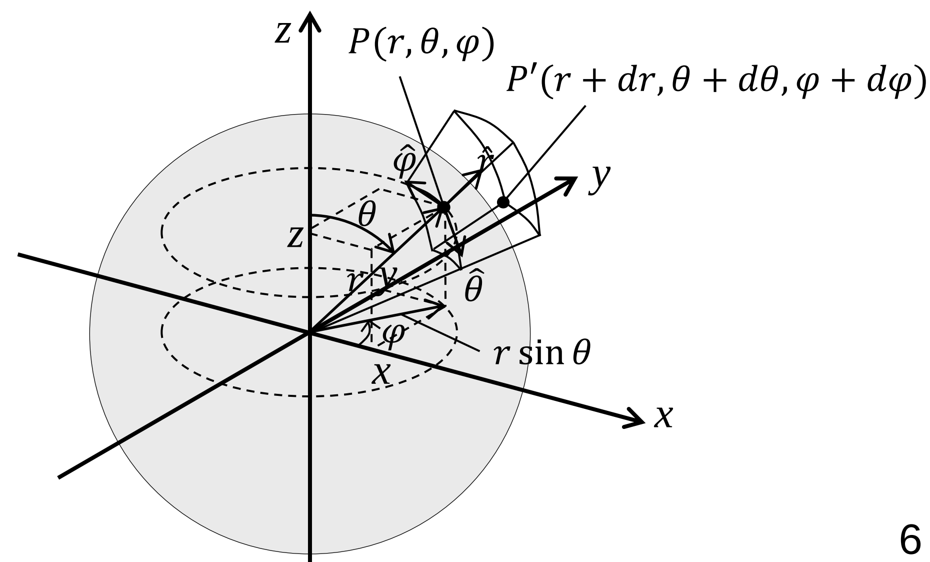
$$\iint_S \hat{r}\cdot d\boldsymbol{S}_r = \iint_S \hat{r}\cdot(\hat{r}r^2\sin\theta\, d\theta d\varphi)$$

$$= \iint_S r^2\sin\theta\, d\theta d\varphi = \int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi} r^2\sin\theta\, d\theta d\varphi$$

$$= r^2\int_{\theta=0}^{\pi}\sin\theta\, d\theta\int_{\varphi=0}^{2\pi} d\varphi = r^2[-\cos\theta]_0^{\pi}\cdot[\varphi]_0^{2\pi}$$

$$= r^2\big(1-(-1)\big)\cdot 2\pi = 4\pi r^2$$





球の体積

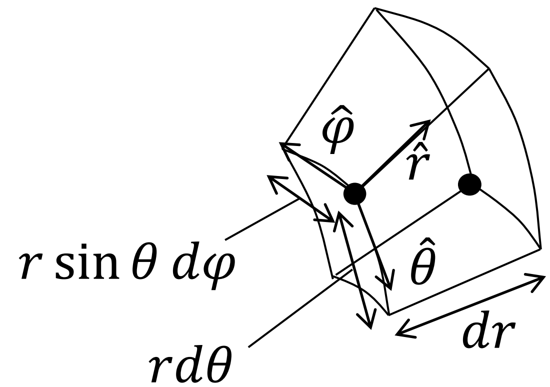
$$\iiint_V dV = \iiint_V r^2 \sin\theta \, dr d\theta d\varphi$$

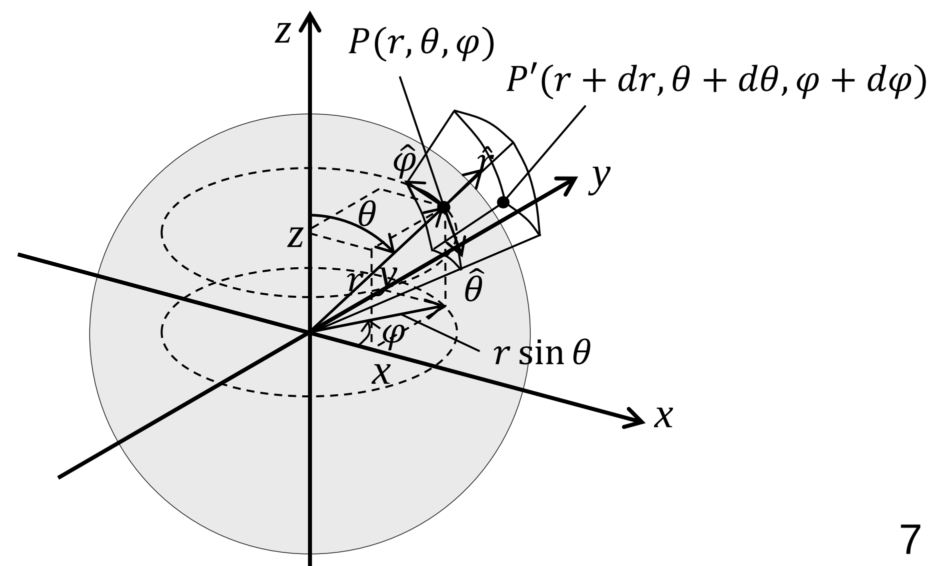
$$= \int_{r=0}^{r} \int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} r^2 \sin\theta \, dr d\theta d\varphi$$

$$= \int_{r=0}^{r} r^2 dr \int_{\theta=0}^{\pi} \sin\theta \, d\theta \int_{\varphi=0}^{2\pi} d\varphi$$

$$= \left[\frac{r^3}{3}\right]_0^r \cdot [-\cos\theta]_0^{\pi} \cdot [\varphi]_0^{2\pi}$$

$$= \frac{r^3}{3}\big(1-(-1)\big) \cdot 2\pi = \frac{4}{3}\pi r^3$$



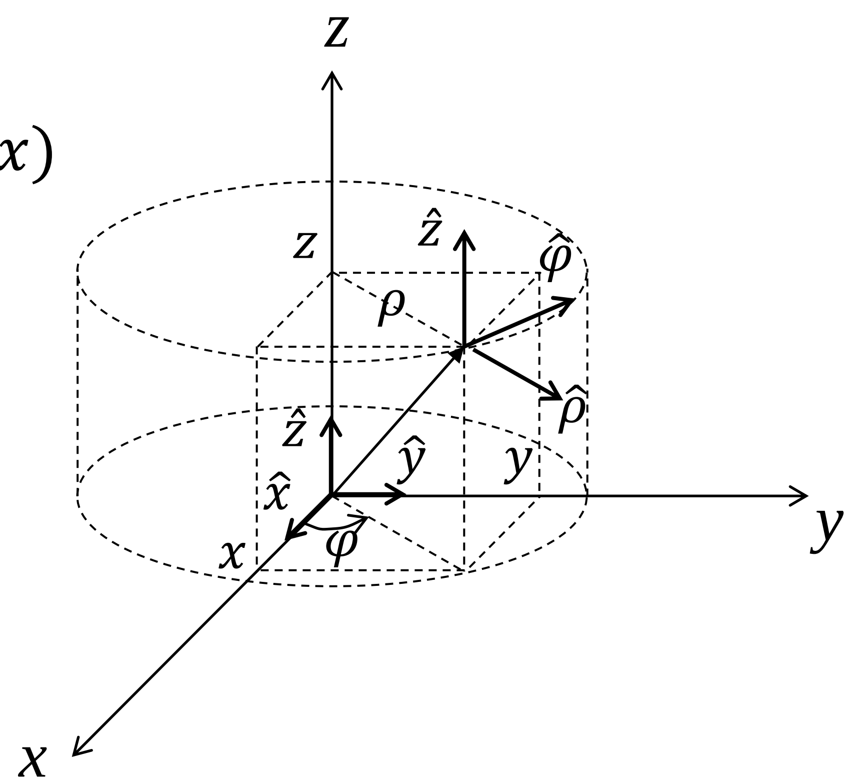


変換(Transform): $(x, y, z) \leftrightarrow (\rho, \varphi, z)$

位置ベクトル(Position Vector)

$$(\rho, \varphi, z) \rightarrow (x, y, z) \quad \begin{cases} x = \rho\cos\varphi \\ y = \rho\sin\varphi \\ z = z \end{cases}$$

$$(x, y, z) \rightarrow (\rho, \varphi, z) \quad \begin{cases} \rho = \sqrt{x^2 + y^2} \\ \varphi = \tan^{-1}(y/x) \\ z = z \end{cases}$$



変換(Transform): $(x, y, z) \leftrightarrow (\rho, \varphi, z)$

ベクトル (Vector)

Calculate inner products of bases.

$$\begin{cases} \hat{x}\cdot\hat{\rho} = \cos\varphi \\ \hat{x}\cdot\hat{\varphi} = -\sin\varphi \\ \hat{x}\cdot\hat{z} = 0 \end{cases} \quad \begin{cases} \hat{y}\cdot\hat{\rho} = \sin\varphi \\ \hat{y}\cdot\hat{\varphi} = \cos\varphi \\ \hat{y}\cdot\hat{z} = 0 \end{cases} \quad \begin{cases} \hat{z}\cdot\hat{\rho} = 0 \\ \hat{z}\cdot\hat{\varphi} = 0 \\ \hat{z}\cdot\hat{z} = 1 \end{cases}$$

Transform (x, y, z) $\boldsymbol{A} = \hat{x}A_x + \hat{y}A_y + \hat{z}A_z$ to (ρ, φ, z) expression.

$$\begin{aligned} \boldsymbol{A} &= \hat{\rho}(\hat{\rho}\cdot\boldsymbol{A}) + \hat{\varphi}(\hat{\varphi}\cdot\boldsymbol{A}) + \hat{z}(\hat{z}\cdot\boldsymbol{A}) \\ &= \hat{\rho}\left(\hat{\rho}\cdot(\hat{x}A_x + \hat{y}A_y + \hat{z}A_z)\right) \\ &\quad +\hat{\varphi}\left(\hat{\varphi}\cdot(\hat{x}A_x + \hat{y}A_y + \hat{z}A_z)\right) \\ &\quad +\hat{z}\left(\hat{z}\cdot(\hat{x}A_x + \hat{y}A_y + \hat{z}A_z)\right) \\ &= \hat{\rho}\left((\hat{\rho}\cdot\hat{x})A_x + (\hat{\rho}\cdot\hat{y})A_y + (\hat{\rho}\cdot\hat{z})A_z\right) \\ &\quad +\hat{\varphi}\left((\hat{\varphi}\cdot\hat{x})A_x + (\hat{\varphi}\cdot\hat{y})A_y + (\hat{\varphi}\cdot\hat{z})A_z\right) \\ &\quad +\hat{z}\left((\hat{z}\cdot\hat{x})A_x + (\hat{z}\cdot\hat{y})A_y + (\hat{z}\cdot\hat{z})A_z\right) \end{aligned}$$

After transformation

Before transformation

Substitute

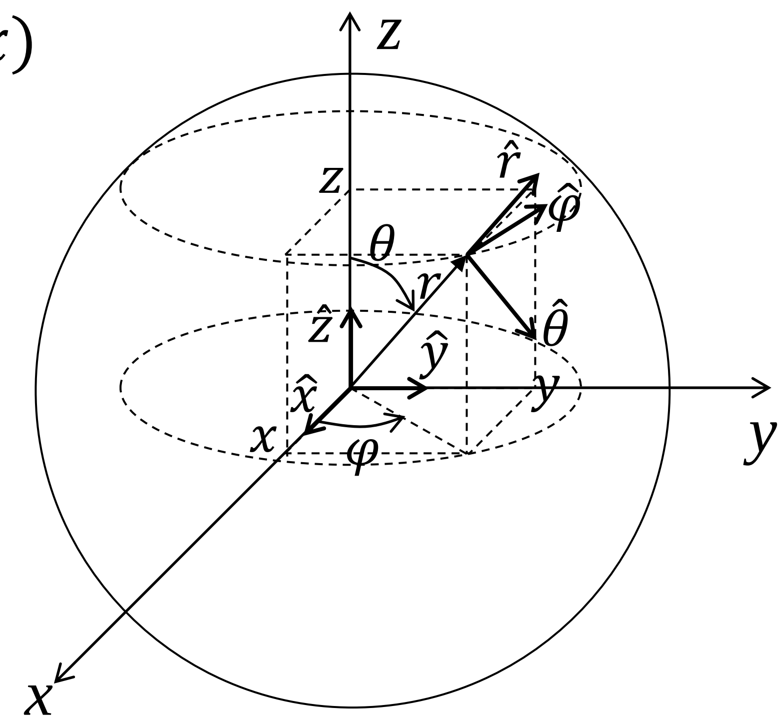
Inverse transformation is the same as well.

変換(Transform): $(x, y, z) \leftrightarrow (r, \theta, \varphi)$

位置ベクトル(Position Vector)

$$(r, \theta, \varphi) \to (x, y, z) \quad \begin{cases} x = r \sin\theta \cos\varphi \\ y = r \sin\theta \sin\varphi \\ z = r \cos\theta \end{cases}$$

$$(x, y, z) \to (r, \theta, \varphi) \quad \begin{cases} r = \sqrt{x^2 + y^2 + z^2} \\ \theta = \cos^{-1}(z/\sqrt{x^2 + y^2 + z^2}) \\ \varphi = \tan^{-1}(y/x) \end{cases}$$



変換(Transform): $(x, y, z) \leftrightarrow (r, \theta, \varphi)$

ベクトル (Vector)

Calculate inner products of bases.

$$\begin{cases} \hat{x}\cdot\hat{r} = \sin\theta\cos\varphi \\ \hat{x}\cdot\hat{\theta} = \cos\theta\cos\varphi \\ \hat{x}\cdot\hat{\varphi} = -\sin\varphi \end{cases} \quad \begin{cases} \hat{y}\cdot\hat{r} = \sin\theta\sin\varphi \\ \hat{y}\cdot\hat{\theta} = \cos\theta\sin\varphi \\ \hat{y}\cdot\hat{\varphi} = \cos\varphi \end{cases} \quad \begin{cases} \hat{z}\cdot\hat{r} = \cos\theta \\ \hat{z}\cdot\hat{\theta} = -\sin\theta \\ \hat{z}\cdot\hat{\varphi} = 0 \end{cases}$$

後の手順は前の資料と同じ。